NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS EXTENSION 1

2018 HSC Course Assessment Task 4 (Trial HSC)
Thursday, 9 August 2018

General instructions

- Working time – 2 hours.
  (plus 5 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt all questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper and hand to examination supervisors.
- A NESA Reference Sheet is provided.

SECTION I

- Mark your answers on the answer grid provided

SECTION II

- Commence each new question on a new booklet. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

NESA STUDENT NUMBER: .... **# BOOKLETS USED:** ....

Class: (please ✓)

○ 12M1 – Mr Sekaran
○ 12M2 – Mrs Bhamra
○ 12M3 – Mr Tan
○ 12M4 – Mrs Gan
○ 12M5 – Mr Lam

Marker's use only

QUESTION	1-10	11	12	13	14	Total	%
MARKS	/10	/15	/15	/15	/15	/70	

Section I

10 marks
Attempt Question 1 to 10
Allow about 15 minutes for this section

Mark your answers on the multiple-choice answer grid

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1. The point P divides the interval from $A(-3,\ 2)$ to $B(4,\ -7)$ externally in the ratio $5:3$. **1**

What is the x-coordinate of P?

(A) $14\frac{1}{2}$ (B) 13 (C) $1\frac{3}{8}$ (D) $-13\frac{1}{2}$

2. Which expression is equal to $\cos(A-B)\cos(A+B)-\sin(A-B)\sin(A+B)$? **1**

(A) $\cos(-2B)$ (B) $\cos 2A$

(C) $2\cos A$ (D) $\pi - \cos 2B$

3. What are the asymptotes of $y = \dfrac{2x^2}{1-x^2}$? **1**

(A) $x = \pm 1$ (B) $x = 2,\ y = \pm 1$

(C) $y = -2,\ x = \pm 1$ (D) $y = 2,\ x = \pm 1$

4. What is the value of $\lim\limits_{x\to 0} \dfrac{\sin\frac{3x}{2}}{6x}$? **1**

(A) $\dfrac{1}{4}$ (B) 4 (C) $\dfrac{1}{2}$ (D) 2

5. What is the derivative of $log_e\left(\dfrac{2x+1}{3x+2}\right)$? **1**

(A) $\dfrac{2}{3}$ (B) $log_e\left(\dfrac{2}{3}\right)$ (C) $\dfrac{1}{(2x+1)(3x+2)}$ (D) $\dfrac{-1}{(2x+1)(3x+2)}$

6. The polynomial $P(x)=x^3+ax^2+2x+1$ has a factor $(x+a)$. **1**

What is the value of a?

(A) $-\frac{1}{2}$ (B) ± 1 (C) $\frac{1}{2}$ (D) $\pm\frac{1}{\sqrt{2}}$

7. What is the general solution of $(sin2x-1)(cos2x-2)=0$? **1**

(A) $x=n\pi+(-1)^n\frac{\pi}{4}$, n is an integer

(B) $x=\frac{n\pi}{2}\pm\frac{3\pi}{4}$, n is an integer

(C) $x=n\pi+(-1)^n\frac{\pi}{2}$, n is an integer

(D) $x=\frac{n\pi}{2}+(-1)^n\frac{\pi}{4}$, n is an integer

8. What is the inverse function of $f(x)=e^{x^3}$? **1**

(A) $f^{-1}(x)=3e^x$

(B) $f^{-1}(x)=3\log_e x$

(C) $f^{-1}(x)=\sqrt[3]{\log_e x}$

(D) $f^{-1}(x)=\sqrt{3\log_e x}$

9. A particle is moving in simple harmonic motion with displacement x. Its velocity v is given by $v^2=9(4-x^2)$. What is the amplitude, A and the period, T of the motion? **1**

(A) $A=3$ and $T=\pi$

(B) $A=3$ and $T=\frac{\pi}{2}$

(C) $A=2$ and $T=\frac{\pi}{3}$

(D) $A=2$ and $T=\frac{2\pi}{3}$

10. What is the value of k such that $\int_2^k \frac{dx}{\sqrt{16-x^2}}=\frac{\pi}{6}$? **1**

(A) $\frac{2\pi}{3}$ (B) $2\sqrt{3}$ (C) $\frac{4\pi}{3}$ (D) $\frac{\sqrt{3}}{2}$

Section II

60 marks
Attempt Questions 11 to 14
Allow about 1 hour and 45 minutes for this section

Write your answers in the writing booklets supplied. Additional writing booklets are available.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) Commence on a NEW booklet **Marks**

(a) Evaluate $\sin^{-1}\left(\cos\dfrac{4\pi}{3}\right)$. **2**

(b) The polynomial equation $x^3 - 3x^2 - 2x + 5 = 0$ has roots $\alpha,\ \beta$ and γ. **2**

. Find the value of $\alpha^2 + \beta^2 + \gamma^2$.

(c) Differentiate $x^2\sin^{-1}2x$ **2**

(d) Using the expansion of $\sin(\theta+\phi)$ prove that $sin75° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$ **2**

(e) Solve the inequality $\dfrac{3}{2x-4} \geq -2$ **3**

(f) (i) Show that $\dfrac{1}{e^x+4e^{-x}} = \dfrac{e^x}{e^{2x}+4}$. **1**

(ii) Hence find $\displaystyle\int \frac{4}{e^x+4e^{-x}}\,dx$ by using the substitution $u = e^x$. **3**

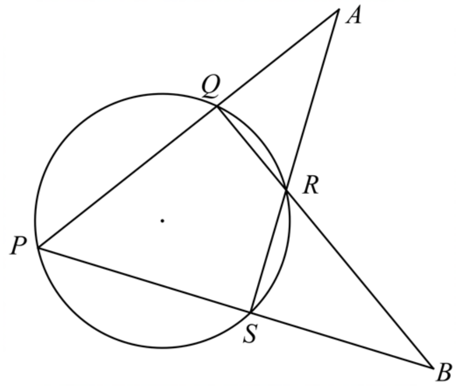
Question 12 (15 Marks) Commence on a NEW booklet **Marks**

(a) One of the roots of the equation $x^3 + kx^2 + 1 = 0$ is the sum of the other two roots.

(i) Show that $x = -\frac{k}{2}$ is a root of the equation **2**

(ii) Find the value of k. **2**

(b) In the diagram, *PQA*, *PSB*, *BRQ* and *SRA* are straight lines and $\angle PAS = \angle PBQ$



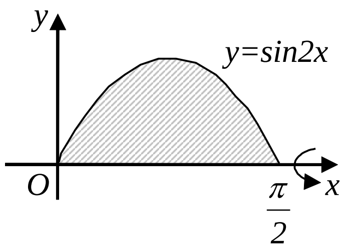
NOT TO SCALE

Copy or trace the diagram into your writing booklet.

(i) Prove that $\angle PQR = \angle PSR$ **2**

(ii) Hence, or otherwise, prove that *PR* is a diameter. **2**

(c) The region bounded by $y = sin2x$ and the x-axis, between $x = 0$ and $x = \frac{\pi}{2}$, is rotated about the x-axis to form a solid. **3**



NOT TO SCALE

Find the volume of the solid.

(d) (i) Starting with the graph of $y = e^{2x}$ show by a sketch that the equation $e^{2x} + 4x - 5 = 0$ has only one solution. **2**

(ii) Taking $x_1 = 0.5$ as a first approximation of the root of the equation $e^{2x} + 4x - 5 = 0$, use one step of Newton's method to find a better approximation correct to 2 decimal places. **2**

Question 13 (15 Marks) Commence on a NEW booklet **Marks**

(a) A sphere is being heated so that its surface area is increasing at a rate of $1.5\ mm^2 s^{-1}$.

Using the formulae for surface area and volume as $A = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$ respectively:

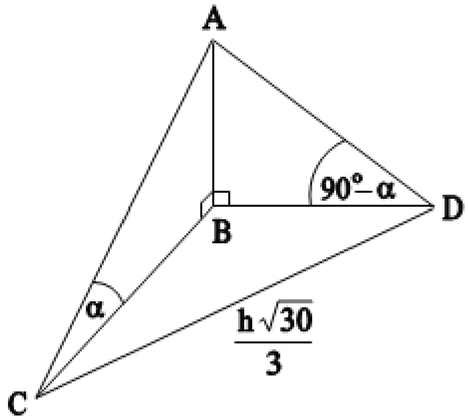
(i) Show that $\frac{dr}{dt} = \frac{3}{16\pi r}$. **2**

(ii) Hence find the rate at which the volume is increasing when the radius is $60\,mm$. **2**

(b) Charles is at point C south of a tower AB of height h metres. His friend Daniel is at a point D, which is closer to the tower and east of it.

The angles of elevation of the top A of the tower from Charles and Daniel's positions are α and $90° - \alpha$ respectively.

The distance CD between Charles and Daniel is $\frac{h\sqrt{30}}{3}$ metres.



NOT TO SCALE

(i) Show that $3tan^4\alpha - 10tan^2\alpha + 3 = 0$ **3**

(ii) Hence, find α, the angle of elevation at which Charles can see the top A of the tower. **2**

(c) Use mathematical induction to prove that for all integers $n \geq 1$ **3**

$$\frac{1}{1\times 4} + \frac{1}{4\times 7} + \ldots + \frac{1}{(3n-2)\,(3n+1)} = \frac{n}{3n+1}$$

Question 13 continued overleaf…

(d) A particle is moving along the x-axis. Its displacement x at time t is given by $x = 2 - 2\sin\left(3t + \frac{\pi}{3}\right)$.

(i) Find an expression for acceleration of the particle in terms of x. **2**

(ii) Hence explain why the motion of this particle is simple harmonic. **1**

Question 14 (15 Marks) Commence on a NEW booklet **Marks**

(a) The rate at which a substance evaporates is proportional to the amount of the substance which has not yet evaporated. That is $\frac{ds}{dt} = -k(s - A)$; where A is the initial amount of substance, s is the amount which has evaporated at time t and k is a constant.

(i) Show that $s = A\left(1 - e^{-kt}\right)$ satisfies the equation $\frac{ds}{dt} = -k(s - A)$ **1**

(ii) Sketch the graph s against t. **2**

(iii) Show that the time it takes for $\frac{7}{8}$ of the substance to evaporate is $\frac{3}{k}\ln 2$. **2**

(b) The acceleration of a particle P is given by $\ddot{x} = 4x(x^2 + 2)$, where x metres is the displacement of P from a fixed point after t seconds. Initially, the particle is at O and has velocity $v = 2\sqrt{2}\ ms^{-1}$.

(i) Show that $v^2 = 2\left(x^2 + 2\right)^2$. **3**

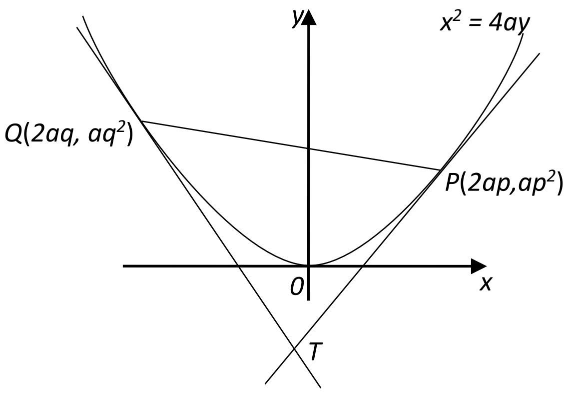
(ii) Explain why the particle will always travel in a positive direction. **1**

(iii) By finding an expression for $\frac{dt}{dx}$, or otherwise, find x as a function of t. **2**

Question 14 continued overleaf…

(c) The points $P\ (2ap, ap^2)$ and $Q\ (2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

The tangents to the parabola at P and Q intersect at the point T. The coordinates of the point T is given by $x = a(p + q)$ and $y = apq$. (Do Not prove this.)



NOT TO SCALE

(i) Show that $p - q = 1 + pq$ if the tangents at P and Q intersect at 45° **2**

(ii) Find the Cartesian equation of the locus of T **2**

End of paper

Normanhurst Boys High School
2018 Higher School Certificate
Trial Examination

Mathematics Extension1
Marking guidelines

Section I

Question	Sample Answer	Marking Key
1	$x = \dfrac{mx_2 + nx_1}{m+n}$ $= \dfrac{3(-3) + -5(4)}{3-5}$ $= \dfrac{-29}{-2}$ $= 14\frac{1}{2}$	**A**
2	$\cos[(A-B)+(A+B)]$ $= \cos 2A$	**B**
3	$1-x^2=0$ $x=\pm 1$ $\lim\limits_{x\to\infty} \dfrac{2x^2}{1-x^2}$ $= \lim\limits_{x\to\infty} \dfrac{2}{\frac{1}{x^2}-1}$ $= \dfrac{2}{-1}$ $= -2$	**C**
4	$\lim\limits_{x\to 0} \dfrac{\sin\frac{3x}{2}}{4\cdot\frac{6x}{4}} = \dfrac{1}{4} \lim \dfrac{\sin\frac{3x}{2}}{\frac{3x}{2}}$ $= \dfrac{1}{4}$	**A**
5	$\dfrac{d}{dx}[\log_e(2x+1) - \log_e(3x+2)]$ $= \dfrac{2}{2x+1} - \dfrac{3}{3x+2}$ $= \dfrac{2(3x+2)-3(2x+1)}{(2x+1)(3x+2)}$ $= \dfrac{1}{(2x+1)(3x+2)}$	**C**

6	$P(-a)=0$, $(-a)^3+a(-a)^2+2(-a)+1=0$ $-a^3+a^3-2a+1=0$ $1=2a$ $\therefore a=\frac{1}{2}$	**C**
7	$\sin 2x=1$, $\cos 2x-2\neq 0$ $2x=n\pi+(-1)^n\frac{\pi}{2}$ $x=\frac{n}{2}\pi+(-1)^n\frac{\pi}{4}$	**D**
8	$x=e^{y^3}$ $\log_e x=y^3$ $\therefore y=\sqrt[3]{\log_e x}$	**C**
9	$v^2=3^2(2^2-x^2)$ $n=3$, $a=2$ $T=\frac{2\pi}{3}$ & $A=2$	**D**
10	$\left[\sin^{-1}\left(\frac{x}{4}\right)\right]_2^k=\frac{\pi}{6}$ $\sin^{-1}\left(\frac{k}{4}\right)-\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$ $\sin^{-1}\left(\frac{k}{4}\right)-\frac{\pi}{6}=\frac{\pi}{6}$ $\sin^{-1}\left(\frac{k}{4}\right)=\frac{\pi}{3}$ $\frac{k}{4}=\sin\left(\frac{\pi}{3}\right)$ $k=4\left(\frac{\sqrt{3}}{2}\right)$ $=2\sqrt{3}$	**B**

Section II

Question 11 (a)

Criteria	Marks
• Provides correct solution	2
• Obtains $\sin^{-1}\left(-\frac{1}{2}\right)$	1

Sample answer

$$\sin^{-1}\left(\cos\frac{4\pi}{3}\right) = \sin^{-1}\left(-\frac{1}{2}\right)$$
$$= -\sin^{-1}\left(\frac{1}{2}\right)$$
$$= -\frac{\pi}{6}$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	2
• Obtains $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha)$	1

Sample answer

$$\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha)$$
$$= (3)^2 - 2(-2)$$
$$= 13$$

Question 11 (c)

Criteria	Marks
• Provides correct solution	2
• Apply product rule correctly	1

Sample answer

$$2x\sin^{-1}2x + \frac{2x^2}{\sqrt{1-4x^2}}$$

Question 11 (d)

Criteria	Marks
• Provides correct solution	2
• Correctly expanded $\sin(30° + 45°)$	1

Sample answer

$$\sin(30°+45°) = \sin 30°\cos 45° + \cos 30°\sin 45°$$
$$\therefore \sin(75°) = \frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}}$$
$$= \frac{1+\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}$$

Question 11 (e)

Criteria	Marks
• Provides correct solution	3
• Achieves $(2x-4)(4x-5) \geq 0$	2
• Multiply both sides by $(2x-4)^2$	1

Sample answer

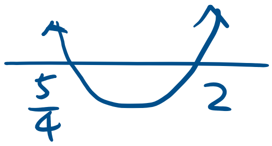
$$(2x-4)^2 \cdot \frac{3}{2x-4} \geq -2(2x-4)^2, \quad x \neq 2$$

$$3(2x-4) \geq -2(2x-4)^2$$

$$(2x-4)[3+2(2x-4)] \geq 0$$

$$2(x-2)(4x-5) \geq 0$$

$$x \leq \frac{5}{4}, \quad x > 2$$



Question 11 (f) (i)

Criteria	Mark
• Provides correct solution	1

Sample answer

$$\frac{1}{e^x+4e^{-x}} = \frac{1}{e^x+\frac{4}{e^x}}$$

$$= \frac{1}{\frac{e^{2x}+4}{e^x}}$$

$$= \frac{e^x}{e^{2x}+4}$$

Question 11 (f) (ii)

Criteria	Marks
• Provides correct solution	3
• Achieves $\int \frac{4}{u^2+4}\,du$	2
• Achieves $du = e^x dx$	1

Sample answer

$$u = e^x$$

$$du = e^x dx$$

$$\int \frac{4}{e^x+4e^{-x}}\,dx = \int \frac{4e^x}{e^{2x}+4}\,dx$$

$$= \int \frac{4}{u^2+4}\,du$$

$$= 4 \cdot \frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)+c$$

$$= 2\tan^{-1}\left(\frac{e^x}{2}\right)+c$$

Question 12 (a) (i)

Criteria	Mark
• Provides correct solution	2
• Achieves $\alpha + \beta + (\alpha + \beta) = -k$	1

Sample answer

Let roots be α, β and $\alpha+\beta$

sum of roots $\alpha+\beta+\alpha+\beta = -k$ ✓

$\alpha+\beta = -\frac{k}{2}$ is a root ✓

Question 12 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Substitute $x = -k/2$ into equation	1

Sample answer

$\left(-\frac{k}{2}\right)^3 + k\left(-\frac{k}{2}\right)^2 + 1 = 0$ ✓

$\frac{-k^3}{8} + \frac{k^3}{4} + 1 = 0$

$-k^3 + 2k^3 + 8 = 0$

$k^3 = -8$

$k = -2$ ✓

Question 12 (b) (i)

Criteria	Mark
• Provides correct solution	2
• Shows exterior angle $\angle PQR$ or $\angle PSR$ in terms of sum of interior angles or prove that $\Delta APS\ similar\ to\ \Delta BPQ$	1

Sample answer

$\angle PAS = \angle PBQ$ (given) —①

$\angle ARQ = \angle BRS$ (vertically opposite angles) —②

$\angle PQR = \angle PAS + \angle ARQ$ (exterior angle of a triangle)

$\angle PSR = \angle PBQ + \angle BRS$ (" " ") ✓

①+② $\angle PAS + \angle ARQ = \angle PBQ + \angle BRS$

$\therefore \angle PQR = \angle PSR$ ✓

Question 12 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Shows $\angle PQR + \angle PSR = 180°$ with reasons	1

Sample answer

$\angle PQR + \angle PSR = 180°$ (opposite angles of cyclic quad.)

From (i) $\angle PQR + \angle PQR = 180°$

$\angle PQR = 90°$

$\therefore$ PR is a diameter (angle in a semicircle is $90°$)

Question 12 (c)

Criteria	Marks
• Provides correct solution	3
• Correct integration	2
• Correct integral for volume in cos4x	1

Sample answer

$$V = \pi \int_0^{\frac{\pi}{2}} \sin^2 2x \, dx$$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{1 - \cos 4x}{2} \, dx$$

$$= \frac{\pi}{2} \left[x - \frac{\sin 4x}{4} \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} \left[\left(\frac{\pi}{2} - \frac{\sin 2\pi}{4} \right) - 0 \right]$$

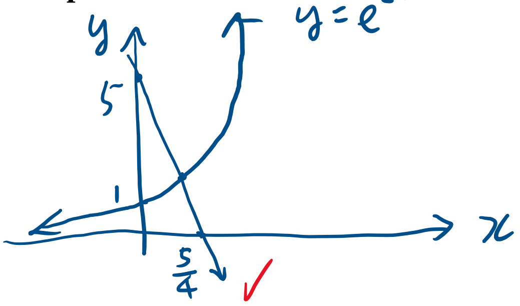
$$= \frac{\pi}{2} \left[\frac{\pi}{2} - 0 \right]$$

$$= \frac{\pi^2}{4} \text{ u}^3$$

Question 12 (d) (i)

Criteria	Marks
• Provides correct solution or correct sketch of $y = e^{2x} + 4x - 5$	2
• Correctly drawn two graphs	1

Sample answer



Graphs of $y = e^{2x}$ and $y = -4x + 5$ intersect only at one point

$\therefore \; e^{2x} = -4x + 5$

ie $e^{2x} + 4x - 5 = 0$ has only one solution

Question 12 (d) (ii)

Criteria	Mark
• Provides correct solution	2
• Correct substitution into formula	1

Sample answer

let $f(x) = e^{2x} + 4x - 5$

$f'(x) = 2e^{2x} + 4$

$x_2 = 0.5 - \dfrac{f(0.5)}{f'(0.5)} = 0.5 - \dfrac{e + 4(0.5) - 5}{2e + 4}$ ✓

$= 0.52985\ldots$

$\doteq 0.53$ (2 d.p.) ✓

Question 13 (a) (i)

Criteria	Marks
• Provides correct solution	2
• Achieves $\dfrac{dA}{dt} = \dfrac{3}{2}$ and $\dfrac{dA}{dr} = 8\pi r$	1

Sample answer

$\dfrac{dA}{dt} = 1.5$ or $\dfrac{3}{2}$ mm s^{-1}

$A = 4\pi r^2 \rightarrow \dfrac{dA}{dr} = 8\pi r$ ✓

$\dfrac{dr}{dt} = \dfrac{dr}{dA} \times \dfrac{dA}{dt}$

$= \dfrac{1}{8\pi r} \times \dfrac{3}{2}$ ✓

$= \dfrac{3}{16\pi r}$

Question 13 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Achieves $\dfrac{dV}{dt} = 4\pi r^2 \times \dfrac{3}{16\pi r}$	1

Sample answer

$V = \dfrac{4}{3}\pi r^3$

$\dfrac{dV}{dr} = 4\pi r^2$

$\dfrac{dr}{dt} = \dfrac{dV}{dr} \times \dfrac{dr}{dt}$

$= 4\pi r^2 \times \dfrac{3}{16\pi r}$

$= \dfrac{3r}{4}$ ✓

when $r = 60$ mm $\dfrac{dV}{dt} = \dfrac{3(60)}{4}$

$= 45$ mm^3 s^{-1} ✓

Question 13 (b) (i)

Criteria	Marks
• Provides correct solution	3
• Correct substitution into Pythagoras' equation	2
• Correctly find *BC* and *BD*	1

Sample answer

$\triangle ABC$, $\tan\alpha = \frac{h}{BC}$ $\rightarrow$ $BC = \frac{h}{\tan\alpha}$

$\triangle ABD$, $\tan(90-\alpha) = \frac{h}{BD}$ $\rightarrow$ $BD = \frac{h}{\tan(90-\alpha)}$

$= \frac{h}{\cot\alpha}$

$= h\tan\alpha$ ✓

$\angle CBD = 90°$

$BC^2 + BD^2 = CD^2$

$\frac{\cancel{h^2}}{\tan^2\alpha} + \cancel{h^2}\tan^2\alpha = \cancel{h^2}\left(\frac{30}{9}\right)$ ✓

$\frac{1}{\tan^2\alpha} + \tan^2\alpha - \frac{10}{3} = 0$

$\times 3\tan^2\alpha$ $\quad 3 + 3\tan^4\alpha - 10\tan^2\alpha = 0$

ie $3\tan^4\alpha - 10\tan^2\alpha + 3 = 0$ ✓

Question 13 (b) (ii)

Criteria	Mark
• Provides correct solution	2
• Correctly solved for two values of $tan\alpha$	1

Sample answer

$(3\tan^2\alpha - 1)(\tan^2\alpha - 3) = 0$

$\tan^2\alpha = \frac{1}{3}$ or 3

$\tan\alpha = \frac{1}{\sqrt{3}}$ or $\sqrt{3}$ only as α is acute

$\alpha = 30°$ or $60°$ ✓

But Daniel is close than Charles to point B

$90 - \alpha > \alpha$

$90 > 2\alpha$

ie $\alpha < 45°$

$\therefore \alpha = 30°$ ✓

Question 13 (c)

Criteria	Marks
• Provides correct proof	3
• Establish induction step, or equivalent merit	2
• Establish initial case, or equivalent merit	1

Sample answer

Let $T_n = \frac{1}{(3n-2)(3n+1)}$, $S_n = \frac{n}{3n+1}$

When $n=1$, LHS $= \frac{1}{1\times4} = \frac{1}{4}$

RHS $= \frac{1}{3+1} = \frac{1}{4}$ $\therefore$ True for $n=1$

Assume true for some integer k ie $S_k = \frac{k}{3k+1}$

RTP $S_{k+1} = \frac{k+1}{3(k+1)+1}$ ie $\frac{k+1}{3k+4}$

Now $S_{k+1} = S_k + T_{k+1}$

$$= \frac{k}{3k+1} + \frac{1}{(3(k+1)-2)(3(k+1)+1)}$$ ✓

$$= \frac{k}{3k+1} + \frac{1}{(3k+1)(3k+4)}$$

$$= \frac{k(3k+4)+1}{(3k+1)(3k+4)}$$

$$= \frac{3k^2+4k+1}{(3k+1)(3k+4)}$$ ✓

$$= \frac{(3k+1)(k+1)}{(3k+1)(3k+4)}$$

$$= \frac{k+1}{3k+4}$$

$\therefore$ True for $n=k+1$

Hence the result is true for all positive integers n by induction.

complete proof ✓

Question 13 (d) (i)

Criteria	Mark
• Provides correct solution	2
• Correct acceleration in terms of *t*	1

Sample answer

$$x = 2 - 2\sin\left(3t + \frac{\pi}{3}\right) \quad — ①$$

$$\dot{x} = -6\cos\left(3t + \frac{\pi}{3}\right)$$

$$\ddot{x} = 18\sin\left(3t + \frac{\pi}{3}\right) \quad ✓$$

$$= 9\left(2\sin\left(3t + \frac{\pi}{3}\right)\right)$$

$$= 9(2 - x) \text{ from ① } \quad ✓$$

Question 13 (d) (ii)

Criteria	Mark
• Provides correct proof	1

Sample answer

$$\ddot{x} = -3^2(x-2)$$

$\therefore$ particle moves in SHM about $x = 2$

Question 14 (a) (i)

Criteria	Mark
• Provides correct proof	1

Sample answer

$$S = A(1 - e^{-kt}) \rightarrow S - A = -Ae^{-kt}$$

$$-(S - A) = Ae^{-kt}$$

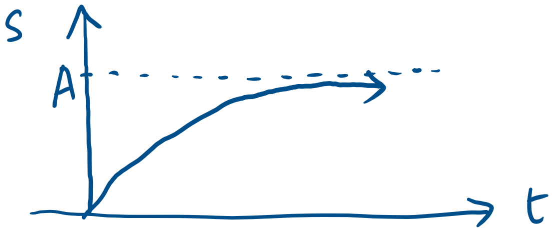
$$\frac{dS}{dt} = Ake^{-kt}$$

$$= -k(S - A)$$

Question 14 (a) (i)

Criteria	Marks
• Provides correct graph	2
• Notes asymptote at *s*=*A* & labelled	1

Sample answer



Question 14 (a) (iii)

Criteria	Marks
• Provides correct solution	2
• Achieves $e^{-kt} = \frac{1}{8}$	1

Sample answer

$$\frac{7}{8}A = A(1 - e^{-kt})$$
$$e^{-kt} = 1 - \frac{7}{8}$$
$$= \frac{1}{8}$$
$$-kt = \ln\left(\frac{1}{8}\right)$$
$$= -\ln(2^3)$$
$$= -3\ln 2$$
$$t = \frac{3\ln 2}{k}$$

Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	3
• Achieves $\frac{1}{2}v^2 = (x^2+2)^2 + c$	2
• Achieves $\frac{1}{2}v^2 = \int 4x(x^2+2)\,dx$	1

Sample answer

$$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = 4x(x^2+2)$$
$$\frac{1}{2}v^2 = \int 4x^3 + 8x\,dx$$
$$= x^4 + 4x^2 + c$$

At $x = 0$, $v = 2\sqrt{2}$

$$\frac{1}{2}(2\sqrt{2})^2 = 0 + c$$
$$\therefore c = 4$$
$$\frac{1}{2}v^2 = x^4 + 4x^2 + 4$$
$$= (x^2+2)^2$$
$$v^2 = 2(x^2+2)^2$$

Question 14 (b) (ii)

Criteria	Mark
• Provides correct reasoning	1

Sample answer

$v^2 = 2(x^2+2)^2$

$v = \pm\sqrt{2}(x^2+2)$

as $v = 2\sqrt{2}$ when $x = 0$

$v = \sqrt{2}(x^2+2) > 0 \quad \forall x$

$\therefore$ the particle will always travel in a positive direction

Question 14 (b) (iii)

Criteria	Marks
• Provides correct solution	2
• Achieves $t = \int \frac{dx}{\sqrt{2}(x^2+2)}$	1

Sample answer

$v = \pm\sqrt{2}(x^2+2)$

$x = 0, \ v = 2\sqrt{2}$

$\therefore \ v = \sqrt{2}(x^2+2)$

$\frac{dx}{dt} = \sqrt{2}(x^2+2)$

$\frac{dt}{dx} = \frac{1}{\sqrt{2}(x^2+2)}$

$t = \frac{1}{\sqrt{2}} \int \frac{dx}{x^2+2}$ ✓

$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right] + c$

$= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + c$

when $t = 0, \ x = 0 \quad \therefore c = 0$

$t = \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$

$\tan(2t) = \frac{x}{\sqrt{2}}$

$x = \sqrt{2}\tan(2t)$ ✓

Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Correctly substitute *p* and *q* into equation $tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$	1

Sample answer

gradient of tangent at P, $m_1 = p$
gradient of tangent at Q, $m_2 = q$

$$\frac{p-q}{1+pq} = \tan 45^\circ$$
$$= 1$$
$$p - q = 1 + pq$$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Achieves equation in $p+q$ and pq	1

Sample answer

From (i) $(p-q)^2 = (1+pq)^2$

$$p^2 + q^2 - 2pq = 1 + (pq)^2 + 2pq$$
$$p^2 + q^2 + 2pq = 1 + (pq)^2 + 6pq$$
$$(p+q)^2 = 1 + (pq)^2 + 6pq$$
$$\left(\frac{x}{a}\right)^2 = 1 + \left(\frac{y}{a}\right)^2 + 6\left(\frac{y}{a}\right)$$

$\times a^2$
$$x^2 = a^2 + y^2 + 6ay$$
$$x^2 - y^2 = a^2 + 6ay$$